Perfect Modeling and Simulation of Measured Spatio-Temporal Wireless Channels

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Abstract
In recent years, various types of spatial channel sounders have been developed with the aim to measure and to investigate the propagation characteristics of spatio-temporal mobile radio channels in typical macrocell, microcell, and picocell environments. After post-processing the collected data, one usually presents the measured channel characteristics in the form of a so-called delay-Doppler power spectral density (PSD) and a delay-angle PSD. A problem is then to find an analytical channel model and/or a simulation model with the property that both the delay-Doppler PSD and the delay-angle PSD of the model approximate as close as possible the corresponding measured system functions. It will be shown in this paper that an exact and general solution to this problem always exists if the measured system functions are consistent. The proposed fundamental procedure is introduced as the perfect channel modeling approach.

Keywords
Channel modeling, perfect channel models, spatio-temporal wideband mobile radio channels, channel measurements, channel simulators, propagation.

INTRODUCTION
The concept of deterministic channel modeling [1] has recently been extended [2] with respect to spatial selectivity resulting in a new class of wideband fading channel models called spatial deterministic Gaussian uncorrelated scattering (SDGUS) model. For this class of channel models, general closed-form expressions can be derived for all relevant system functions, such as, e.g., the delay-Doppler PSD and the delay-direction (angle) PSD. The obtained formulae reveal that these two system functions are completely determined by the parameters of the SDGUS model. This property forms the basis for the fundamental proof that all types of measured delay-Doppler-direction characteristics, provided that the measured delay-Doppler and delay-direction PSDs are consistent. A necessary but not sufficient condition for consistency is that the delay profile derived from the measured delay-Doppler PSD is identical to that which can be obtained from the measured delay-direction PSD. For non-consistent measurement results, a slight modification of the perfect channel modeling approach is proposed. It will be shown that all parameters of the SDGUS model can be extracted unambiguously from the measurement results.

A channel model that is perfectly fitted to measurements is introduced as perfect spatio-temporal channel model, and analogously the therefrom derivable channel simulator is called a perfect spatio-temporal channel simulator. Perfect spatio-temporal channel simulators enable the emulation of measured direction-dispersive wideband mobile radio channels without producing any model error or making approximations. Such a simulator is of central importance for testing, optimizing, and studying the performance of future wireless systems with smart antennas under real-world propagation conditions.

THE SDGUS MODEL
In this section, we briefly review the SDGUS model introduced in [2]. The time-space-variant impulse response \( \tilde{h}(t', t, x) \) of the SDGUS model can be represented by a sum of \( \mathcal{L} \) discrete propagation paths with different propagation delays as follows

\[
\tilde{h}(t', t, x) = \sum_{\ell=0}^{\mathcal{L}-1} \tilde{a}_\ell \tilde{\mu}_\ell(t, x) \delta(t' - \tilde{\tau}_\ell) \quad (1)
\]

where \( \tilde{a}_\ell \) denotes the path gain of the \( \ell \)th propagation path, and \( \tilde{\tau}_\ell \) is the corresponding discrete propagation delay. In (1), the disturbances caused by the Doppler effect and the spatial behavior of the channel are modeled by spatio-temporal complex deterministic Gaussian processes \( \tilde{\mu}_\ell(t, x) \) of the form

\[
\tilde{\mu}_\ell(t, x) = \sum_{n=1}^{N_\ell} c_{n,\ell} e^{j(2\pi f_{n,\ell} t + \theta_{n,\ell})} e^{j2\pi \lambda_0^{-1} \Omega_{n,\ell} x} \quad (2)
\]

where \( \ell = 0, 1, \ldots, \mathcal{L} - 1 \). Here, \( N_\ell \) represents the number of exponential functions assigned to the \( \ell \)th propagation path, \( c_{n,\ell} \) is the Doppler coefficient of the \( n \)th component of the \( \ell \)th propagation path, and \( f_{n,\ell} \) and \( \theta_{n,\ell} \) are the corresponding discrete Doppler frequency and phase, respectively. The symbol \( \lambda_0 \) denotes the wavelength, and \( \Omega_{n,\ell} \) is called the incidence direction, which is related with the azimuth angle of arrival \( \phi_{n,\ell} \) via

\[
\Omega_{n,\ell} = \sin \phi_{n,\ell} \quad . (3)
\]
In the SDGUS model, the parameters $L$, $N_t$, $\tilde{a}_t$, $\tilde{\tau}_t$, $c_{n,t}$, $f_{n,t}$, and $\Omega_{n,t}$ are constant quantities, which can be determined from measurements as described below. In (2), the phases $\theta_{n,t}$ are considered as outcomes of a random generator having a uniform distribution within the interval $[0, 2\pi)$. From this point of view, we may regard the model parameters appearing in (1) and (2) as known and constant quantities. Hence, the time-space-variant impulse response $\tilde{h}(\tau', t, x)$ is a completely deterministic function. Therefore, the correlation properties of $\tilde{h}(\tau', t, x)$ have to be derived from time averages rather than from statistical averages.

Next, we impose the uncorrelated scattering (US) condition on our model. This requires that any couple of complex deterministic Gaussian processes $\tilde{\mu}_h(t, x)$ and $\tilde{\mu}_a(t, x)$ must be uncorrelated for different propagation paths $\tilde{\tau}_t$ and $\tilde{\tau}_a$ with $\ell \neq \lambda$, where $\ell, \lambda = 0, 1, \ldots, \mathcal{L} - 1$. This condition is fulfilled if, and only if, the sets of discrete Doppler frequencies $\{f_{n,t}\}_{n=1}^{N_t}$ and $\{f_{n,\lambda}\}_{n=1}^{N_\lambda}$ are mutually disjoint for different propagation paths. Hence, the US condition can be expressed as follows:

US $\iff \tilde{\mu}_h(t, x)$ and $\tilde{\mu}_a(t, x)$ are uncorrelated if $\ell \neq \lambda$ \hspace{1cm} (4a)

US $\iff \{f_{n,t}\}_{n=1}^{N_t} \cap \{f_{n,\lambda}\}_{n=1}^{N_\lambda} = \{0\}$ if $\ell \neq \lambda$ \hspace{1cm} (4b)

where $\ell, \lambda = 0, 1, \ldots, \mathcal{L} - 1$. If the US condition is fulfilled, then the delay-Doppler-direction PSD, $\tilde{S}(\tau', f, \Omega)$, of the SDGUS model is given by (without proof)

$$\tilde{S}(\tau', f, \Omega) = \sum_{\ell=0}^{\mathcal{L}-1} \sum_{n=1}^{N_t} (\tilde{a}_t c_{n,t})^2 \cdot \delta(\tau' - \tilde{\tau}_t) \delta(f - f_{n,t}) \delta(\Omega - \Omega_{n,t}) .$$

Without loss of generality, we can assume that the SDGUS model is normalized so that $\iint \tilde{S}(\tau', f, \Omega) df \, d\Omega = 1$. Such a normalized model is obtained by imposing on the path gains $\tilde{a}_t$ and the Doppler coefficients $c_{n,t}$ the following marginal conditions:

$$\sum_{\ell=0}^{\mathcal{L}-1} \tilde{a}_t^2 = 1 , \quad \sum_{n=1}^{N_t} c_{n,t}^2 = 1$$ \hspace{1cm} (6a,b)

for $\ell = 0, 1, \ldots, \mathcal{L} - 1$.

Let us assume that the mobile station’s antenna is omnidirectional and that the base station is equipped with a uniform linear antenna array consisting of $M$ antenna elements separated in $x$-direction by the antenna spacing $\Delta$. Then, the time-space-variant impulse response related to the $m$th antenna element is given by $\tilde{h}_m(\tau', t, x) = \tilde{h}(\tau', t, x = \Delta(m - 1))$. The implementation of $\tilde{h}_m(\tau', t, x)$ can easily be performed on a computer or a hardware platform, and, thus, enables the emulation of the received signal at the $m$th antenna of spatio-temporal mobile radio channels.

### Some Further Important System Functions of SDGUS Models

In this subsection, we discuss some further important system functions to give insight into the statistical properties of SDGUS models. Especially, the delay-Doppler PSD and the delay-direction PSD are of central importance for the intention of this paper.

**Delay-Doppler PSD**

The delay-Doppler PSD, denoted as $\tilde{S}_{\tau'f}(\tau', f)$, is obtained by integrating the delay-Doppler-direction PSD $\tilde{S}(\tau', f, \Omega)$ over the incidence directions $\Omega$, i.e.,

$$\tilde{S}_{\tau'f}(\tau', f) := \int_{-\infty}^{\infty} \tilde{S}(\tau', f, \Omega) d\Omega .$$

Substituting (5) into (7) enables the presentation of $\tilde{S}_{\tau'f}(\tau', f)$ in a closed form given by

$$\tilde{S}_{\tau'f}(\tau', f) = \sum_{\ell=0}^{\mathcal{L}-1} \sum_{n=1}^{N_t} (\tilde{a}_t c_{n,t})^2 \delta(\tau' - \tilde{\tau}_t) \delta(f - f_{n,t}) .$$

This result shows that the delay-Doppler PSD $\tilde{S}_{\tau'f}(\tau', f)$ consists of $\sum_{\ell=0}^{\mathcal{L}-1} N_t$ delta functions located in the $(\tau', f)$-plane at $(\tilde{\tau}_t, f_{n,t})$ and weighted by $(\tilde{a}_t c_{n,t})^2$. Obviously, $\tilde{S}_{\tau'f}(\tau', f)$ is completely determined by the model parameters $L$, $N_t$, $\tilde{a}_t$, $\tilde{\tau}_t$, $c_{n,t}$, and $f_{n,t}$.

**Delay-Direction PSD**

The delay-direction PSD, denoted as $\tilde{S}_{\tau\Omega}(\tau', \Omega)$, is obtained by integrating the delay-Doppler-direction PSD $\tilde{S}(\tau', f, \Omega)$ over the Doppler frequencies $f$, i.e.,

$$\tilde{S}_{\tau\Omega}(\tau', \Omega) := \int_{-\infty}^{\infty} \tilde{S}(\tau', f, \Omega) df .$$

After substituting (5) into (9), we can present $\tilde{S}_{\tau\Omega}(\tau', \Omega)$ in the following closed form

$$\tilde{S}_{\tau\Omega}(\tau', \Omega) = \sum_{\ell=0}^{\mathcal{L}-1} \sum_{n=1}^{N_t} (\tilde{a}_t c_{n,t})^2 \delta(\tau' - \tilde{\tau}_t) \delta(f - \Omega_{n,t}) .$$

Note that the delay-direction PSD $\tilde{S}_{\tau\Omega}(\tau', \Omega)$ is composed of a sum of $\sum_{\ell=0}^{\mathcal{L}-1} N_t$ delta functions located in the $(\tau', \Omega)$-plane at $(\tilde{\tau}_t, \Omega_{n,t})$ and weighted by $(\tilde{a}_t c_{n,t})^2$. Thus, the behavior of $\tilde{S}_{\tau\Omega}(\tau', \Omega)$ is completely determined by the model parameter $L$, $N_t$, $\tilde{a}_t$, $\tilde{\tau}_t$, $c_{n,t}$, and $\Omega_{n,t}$.

**Delay Profile**

The delay profile $\tilde{S}_{\tau'}(\tau')$ can be obtained from the delay-Doppler PSD $\tilde{S}_{\tau'f}(\tau', f)$ using (8), or the delay-direction PSD $\tilde{S}_{\tau\Omega}(\tau', \Omega)$ using (10), according to

$$\tilde{S}_{\tau'}(\tau') := \int_{-\infty}^{\infty} \tilde{S}_{\tau'f}(\tau', f) df = \int_{-\infty}^{\infty} \tilde{S}_{\tau\Omega}(\tau', \Omega) d\Omega$$

$$= \sum_{\ell=0}^{\mathcal{L}-1} \tilde{a}_t^2 \delta(\tau' - \tilde{\tau}_t) .$$

It should be observed that the delay profile obtained from the delay-Doppler PSD is identical to the delay profile derived
from the delay-direction PSD. A channel model with this property is said to be consistent with respect to the delay profile.

**THE PERFECT CHANNEL MODELING APPROACH**

In this section, an approach is described that enables the fitting of both the delay-Doppler PSD $\tilde{S}_{\tau,f}(\tau', f)$ and the delay-direction PSD $\tilde{S}_{\tau,\Omega}(\tau', \Omega)$ of the SDGUS model to any given measured delay-Doppler PSD $S_{\tau,f}^*(\tau', f)$ and measured delay-direction PSD $S_{\tau,\Omega}^*(\tau', \Omega)$, respectively. The proposed procedure can always be applied, if the measured system functions $S_{\tau,f}^*(\tau', f)$ and $S_{\tau,\Omega}^*(\tau', \Omega)$ are consistent and discrete with respect to the variables $\tau'$, $f$, and $\Omega$. A channel model, the delay-Doppler-direction PSD $\tilde{S}(\tau', f, \Omega)$ of which is identical to the measured delay-Doppler-direction PSD $S^*(\tau', f, \Omega)$, i.e.,

$$\tilde{S}(\tau', f, \Omega) = S^*(\tau', f, \Omega)$$

is called a perfect channel model. Due to the relations (7) and (9), a perfect channel model fulfills the conditions

$$\tilde{S}_{\tau,f}(\tau', f) = S_{\tau,f}^*(\tau', f), \quad \tilde{S}_{\tau,\Omega}(\tau', \Omega) = S_{\tau,\Omega}^*(\tau', \Omega).$$

(13a,b)A semi-perfect channel model is characterized by

$$\tilde{S}_{\tau,f}(\tau', f) = S_{\tau,f}^*(\tau', f), \quad \tilde{S}_{\tau,\Omega}(\tau', \Omega) \approx S_{\tau,\Omega}^*(\tau', \Omega)$$

or, alternatively,

$$\tilde{S}_{\tau,f}(\tau', f) \approx S_{\tau,f}^*(\tau', f), \quad \tilde{S}_{\tau,\Omega}(\tau', \Omega) = S_{\tau,\Omega}^*(\tau', \Omega).$$

(14a,b)

In the following, we describe a procedure for the computation of the model parameters $\mathcal{L}$, $\{N_l\}$, $\{\tilde{a}_l\}$, $\{\bar{\tau}_l\}$, $\{c_{n,\ell}\}$, $\{f_{n,\ell}\}$, and $\{\Omega_{n,\ell}\}$ of the SDGUS model in such a way that (13) or (14) is fulfilled.

**Fitting the Delay-Doppler PSD**

Since it has been assumed that the measured delay-Doppler PSD $S_{\tau,f}^*(\tau', f)$ is discrete in $\tau'$ and $f$-direction, we can alternatively represent $S_{\tau,f}^*(\tau', f)$ by a so-called delay-Doppler matrix

$$S_{\tau,f}^* = \begin{pmatrix}
    s_{1,0}^* & s_{1,1}^* & \cdots & s_{1,L-1}^* \\
    s_{2,0}^* & s_{2,1}^* & \cdots & s_{2,L-1}^* \\
    \vdots & \vdots & \ddots & \vdots \\
    s_{N,0}^* & s_{N,1}^* & \cdots & s_{N,L-1}^* \\
    0 & 0 & \cdots & 0
\end{pmatrix}$$

with $N$ rows and $L$ columns. Without loss of generality, we assume that the $l_1$ norm of the matrix $S_{\tau,f}^*$ is equal to one, i.e., $\|S_{\tau,f}^*\|_1 = 1$. Let $\Delta f$ and $\Delta \tau'$ be the resolution of the channel sounder in $f$- and $\tau'$-direction, respectively, then the number of rows $N$ and the number of columns $L$ can directly be related to the measured maximum Doppler frequency $f_{\text{max}}$ and the maximum propagation delay $\tau'_{\text{max}}$ as follows

$$N = \frac{2f_{\text{max}}}{\Delta f} + 1 \quad \text{and} \quad L = \frac{\tau'_{\text{max}}}{\Delta \tau'} + 1.$$  (17a,b)

The row index $n$ and the column index $l$ are related to the measured discrete Doppler frequency $f_{n,\ell}$ and the measured propagation delay $\tau'_{n,\ell}$ as follows:

$$f_{n,\ell} = -f_{\text{max}} + \Delta f \cdot (n - 1)$$

(18a)

$$\tau'_{n,\ell} = \Delta \tau' \cdot (l - 1)$$

(18b)

where $l = 1, 2, \ldots, L$ and $n = 1, 2, \ldots, N$. Now, we are in a position to determine the parameters of the SDGUS model in such a way that the identity (13a) holds. For that purpose, the number of exponential functions $N_l$ of the deterministic Gaussian processes $\tilde{a}_l(t, x)$ [see (2)] will be identified with the number of rows of $S_{\tau,f}^*$. Therefore, we define $N_l := N$ for all $l = 0, 1, \ldots, \mathcal{L} - 1$. Furthermore, the number of discrete propagation paths $\mathcal{L}$ must be in accordance with the number of columns of $S_{\tau,f}^*$. To assure this, we define $\mathcal{L} := L$. Moreover, the definitions $f_{n,\ell} := f_{n,\ell}^*$ and $\tau'_{n,\ell} := \tau'_{n,\ell}^*$ guarantee that the discrete Doppler frequencies and the discrete propagation delays of the SDGUS model are exactly identical to the respective measured quantities.

Now, the delay-Doppler PSD $\tilde{S}_{\tau,f}(\tau', f)$ [see (8)] of the SDGUS model can alternatively be presented by the matrix

$$\tilde{S}_{\tau,f} = \begin{pmatrix}
    \tilde{s}_{1,0} & \tilde{s}_{1,1} & \cdots & \tilde{s}_{1,L-1} \\
    \tilde{s}_{2,0} & \tilde{s}_{2,1} & \cdots & \tilde{s}_{2,L-1} \\
    \vdots & \vdots & \ddots & \vdots \\
    \tilde{s}_{N,0} & \tilde{s}_{N,1} & \cdots & \tilde{s}_{N,L-1} \\
    0 & 0 & \cdots & 0
\end{pmatrix}$$

$$f_{\text{max}}$$

(19)

where the entries $\tilde{s}_{n,\ell}$ are given by $\tilde{s}_{n,\ell} = (\tilde{a}_l c_{n,\ell})^2$. The remaining model parameters influencing the behavior of (8) can readily be obtained by imposing on the SDGUS model that the entries of $\tilde{S}_{\tau,f}$ must be identical to the entries of the measured delay-Doppler matrix $S_{\tau,f}^*$, i.e., $\tilde{s}_{n,\ell} = (\tilde{a}_l c_{n,\ell})^2 = s_{n,\ell}^*$. From this relation and (6b), we find directly:

$$\tilde{a}_l = \sum_{n=1}^{N} s_{n,\ell}^* \tilde{a}_l$$

(20a)

$$c_{n,\ell} = \sqrt{s_{n,\ell}^* / \tilde{a}_l}$$

(20b)

for $\ell = 0, 1, \ldots, \mathcal{L} - 1$ (L - 1) and $n = 1, 2, \ldots, N_l (N)$. Since $N_l := N$, $\mathcal{L} := L$, $f_{n,\ell} := f_{n,\ell}^*$, $\tau'_{n,\ell} := \tau'_{n,\ell}^*$, and $\tilde{s}_{n,\ell} = s_{n,\ell}^*$, it follows obviously that the desired relation $\tilde{S}_{\tau,f} = S_{\tau,f}^*$ holds, which is equivalent to (13a). Due to the limited resolution in the Doppler frequency domain, one cannot rule out that some of the measured Doppler frequencies are identical in different propagation paths. In such cases, the US condition (4) is violated. To obtain an
Fitting the Delay-Direction PSD

Based on our assumption that the measured delay-direction PSD $S_{\tau,\Theta}(\tau', \Theta)$ is given in a discrete form, we can alternatively represent $S_{\tau,\Theta}(\tau', \Theta)$ by the matrix

$$S_{\tau,\Theta} = \begin{pmatrix} r_{1,0} & r_{1,1} & \cdots & r_{1,L-1} \\ r_{2,0} & r_{2,1} & \cdots & r_{2,L-1} \\ \vdots & \vdots & \ddots & \vdots \\ r_{M,0} & r_{M,1} & \cdots & r_{M,L-1} \end{pmatrix} = \begin{pmatrix} -\Omega_{\text{max}} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{pmatrix} \Omega \end{pmatrix}$$

(21)

which is called the delay-direction matrix. This matrix has $M$ rows and $L$ columns, where $M$ is allowed to be different from the number of rows of $S_{\tau,f}$. Without loss of generality, we assume that the $l_1$ norm of the matrix $S_{\tau,\Theta}$ is equal to one, i.e., $\|S_{\tau,\Theta}\|_1 = 1$. The row index $m$ refers to the measured discrete direction

$$\Omega_{m,\ell} = -\Omega_{\text{max}} + \Delta \Omega \cdot (m - 1)$$

(22)

where $m = 1, 2, \ldots, M$. The quantity $\Omega_{\text{max}}$ denotes the largest measured direction, and $\Delta \Omega$ is the resolution of the channel sounder in $\Omega$-direction.

If the measured delay-direction PSD $S_{\tau,\Theta}(\tau', \Theta)$ and the measured delay-Doppler PSD $S_{\tau,f}(\tau', f)$ are consistent, then the corresponding matrices $S_{\tau,\Theta}$ and $S_{\tau,f}$ have some interesting properties:

(i) The path gains $\tilde{a}_\ell$ obtained from (16) and (21) are identical and given by

$$\tilde{a}_\ell = \left( \sum_{n=1}^{N} s_{n,\ell} \right)^{1/2} = \left( \sum_{m=1}^{M} r_{m,\ell}^* \right)^{1/2}$$

(23)

(ii) The entries of the $t$th column of $S_{\tau,f}$ may differ from the entries of $l$th column of $S_{\tau,\Theta}$ only by permutation.

In case the condition (ii) is fulfilled, then, for any value of $n = 1, 2, \ldots, N$, an integer $m \in \{1, 2, \ldots, M\}$ can be found so that $r_{m,\ell} = s_{n,\ell}^*$ holds. With the knowledge of the index $m$, we can determine $\Omega_{m,\ell}$ using (22). The desired discrete direction $\Omega_{m,\ell}$ of the SDGUS model is then given by identifying $\Omega_{m,\ell}$ with $\Omega_{m,\ell}$, i.e., $\Omega_{m,\ell} = \Omega_{m,\ell}^*$. The remaining parameters $\tilde{\tau}'_f$, $\tilde{a}_\ell$, and $c_{n,\ell}$ can be computed by using (18b), (20a), and (20b), respectively. Hence, all model parameters describing the delay-direction PSD $S_{\tau,\Theta}(\tau', \Theta)$ can be determined in such a way that $S_{\tau,\Theta}(\tau', \Theta)$ is perfectly fitted to the measured delay-direction PSD $S_{\tau,\Theta}^*(\tau', \Theta)$. Obviously, $S_{\tau,\Theta}(\tau', \Theta)$ is consistent with $S_{\tau,f}(\tau', f)$, because both system functions are depending on the same parameters $L$, $\{N_{\ell}\}$, $\{\tilde{a}_\ell\}$, $\{\tilde{\tau}'_f\}$, and $\{c_{n,\ell}\}$.

Non-Perfect Fitting

Due to measurement errors, however, the measured delay-direction PSD $S_{\tau,\Theta}^*(\tau', \Theta)$ is in general not consistent with the measured delay-Doppler PSD $S_{\tau,f}^*(\tau', f)$. In such cases, where (i) and especially (ii) are not fulfilled, only semi-perfect channel models can be derived.

For reasons of brevity, we focus our attention on the development of semi-perfect channel models described by (14). The relation (14a) can be fulfilled by applying the techniques described in the previous subsection. To find a solution for (14b), we proceed as follows. If (ii) is not fulfilled, then, for any measured value $s_{n,\ell}^*$, we have to find the entry $r_{m,\ell}$ which is closest in value to $s_{n,\ell}^*$. Here, the index $\lambda$ refers to the $\lambda$th propagation path and ranges from $\ell - \ell_0$ to $\ell + \ell_0$, where $\ell_0$ denotes a sufficiently large integer.

Our next task is to construct an auxiliary delay-direction matrix $S_{\tilde{\tau},\Theta}$ containing the $n$th row and the $\ell$th column the entry $s_{n,\ell}^*$ of $S_{\tau,f}$ which was selected as the best approximation of the entry $r_{m,\lambda}$ of the measured delay-direction matrix $S_{\tau,\Theta}$. The resulting auxiliary delay-direction matrix $S_{\tilde{\tau},\Theta}$ is consistent with $S_{\tau,f}$ and can now be used as starting point for the perfect fitting procedure described in the previous subsubsection.

APPLICATION TO MEASUREMENTS

In what follows, we apply the described procedure to real-world measurement data, which have been collected in industrial indoor buildings by using a channel sounder with a center frequency of 5.2 GHz and a bandwidth of 120 MHz [3]. An example for a measured delay-Doppler PSD $S_{\tau,f}(\tau', f)$ is illustrated in Figure 1. The corresponding measured delay-angle PSD $S_{\tau,\phi}(\tau', \phi)$, which is equivalent to the delay-direction PSD $S_{\tau,\Theta}(\tau', \Theta)$, is shown in Figure 2. Determining from $S_{\tau,f}(\tau', f)$ the model parameters $L$, $\{N_{\ell}\}$, $\{\tilde{a}_\ell\}$, $\{\tilde{\tau}'_f\}$, $\{c_{n,\ell}\}$, and $\{f_{n,\ell}\}$ by applying the perfect channel modeling approach and putting the obtained parameters in (8) results in the delay-Doppler PSD $S_{\tau,\Theta}(\tau', f)$ of the SDGUS model presented in Figure 3. It should be observed that the measured delay-Doppler PSD $S_{\tau,f}(\tau', f)$ (see Figure 1) is identical to the modeled delay-Doppler PSD $S_{\tau,\Theta}(\tau', f)$ (see Figure 3). The non-consistency of the measured system functions $S_{\tau,f}(\tau', f)$ and $S_{\tau,\Theta}(\tau', \Theta)$ prevents the design of a perfect channel model. However, a semi-perfect channel model can be derived by using the techniques described above. Figure 4 presents the obtained delay-angle PSD $S_{\tau,\phi}(\tau', \phi)$ of the resulting semi-perfect SDGUS model. A comparison between Figure 4 and Figure 2 shows that $S_{\tau,\phi}(\tau', \phi)$ closely approximates $S_{\tau,\Theta}(\tau', \Theta)$.

CONCLUSION

In this paper, an approach has been introduced for the design of perfect channel models. The proposed procedure enables to find the parameters of SDGUS models in such a way that both the delay-Doppler PSD and the delay-direction PSD of the SDGUS model are identical to the corresponding mea-
measured system functions of real-world spatio-temporal wideband mobile radio channels. For a successful application of the perfect channel modeling approach, it is important that the measured delay-Doppler PSD is consistent with the corresponding measured delay-direction PSD. Due to measurement errors, however, the consistency condition is in general not fulfilled. For non-consistent measurements, a modified procedure has been proposed enabling the development of semi-perfect channel models. A semi-perfect channel model has a delay-Doppler PSD which is perfectly fitting to the measured delay-Doppler PSD, whereas the delay-direction PSD approximates as close as possible the measured delay-direction PSD. Since the SDGUS model is based on sums of exponential functions with constant parameters, the proposed procedure can be used for the design of efficient spatio-temporal channel simulators enabling the emulation of real-world spatio-temporal wideband mobile radio channels.

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